The thermal conductivity as a function of temperature, calculated by the methods described above, is shown in Fig. 2. The comparison of these thermal conductivities with those determined for the same materials by monotonic heating at temperatures below the thermal decomposition region showed a reasonable agreement.

NOTATION

τ, time; x, thickness coordinate; δ, sample thickness; x_{bd}, isothermal coordinate of the beginning of decomposition, $t(x, \tau)$, calculated temperature field in the specimen; $u(x, \tau)$, experimentally determined temperature field in the specimen; $t_0(x)$, initial temperature profile in the specimen; χ degree of thermal decomposition; ρ_0, initial density of the material; ρ_2, instantaneous density of the solid residue; $c_1(t)$, specific heat of the volatile thermal-decomposition products; $c_2(t)$ and $\lambda(t)$, specific heat and thermal conductivity of the material; K_{mc}, mass concentration of the solid residue in the thermal decomposition products; Q, calorific effect of thermal decomposition at the temperature of decomposition; Δ, root-mean-square error of the temperature measurements; f, porosity of the coke; τ^*, duration of the thermal treatment. Indices: $k = 1, 2, \ldots, K$ is number of the approximation interval, and $i = 0, 1, 2, \ldots, I$ is the number of the point through the thickness.

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A MODIFIED METHOD OF DETERMINING THE THERMAL CONSTANTS OF NONMETALLIC MATERIALS

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We describe a method and the experimental set-up for a comprehensive determination of thermal constants. The method is applied to reinforced composite materials.

In the current technology there is some interest in composite materials reinforced by metallic inclusions in the form of a foil or wire. In some cases, a contact zone is formed between the composite material and the inclusion which is different from the reinforcing material.

To find the thermal conductivity of such a system it is necessary to know the thermal constants of the initial materials and of the contact zone. The properties of the initial materials are often known fairly reliably; to determine the properties of the contact zone by the usual methods, however, is usually difficult because the thickness of the zone is small and its separation is practically impossible.

Physical Basis of the Method

In the present combined method of determination of the thermal constants of nonmetallic materials we used as a source of heat and as the temperature sensor a small strip of metallic foil or a small-diameter wire from aluminum, copper, or silver. For the above-mentioned composite materials this element can be the metal inclusion.

The method is based on the combination of the method of regular thermal regime of the third kind [1] and of the cylindrical-probe method [2]. This makes it possible to determine in one experiment the coefficients of thermal activity, thermal diffusivity, and thermal conductivity. The experiment is internally consistent since from any two of the coefficients it is possible to find the third.

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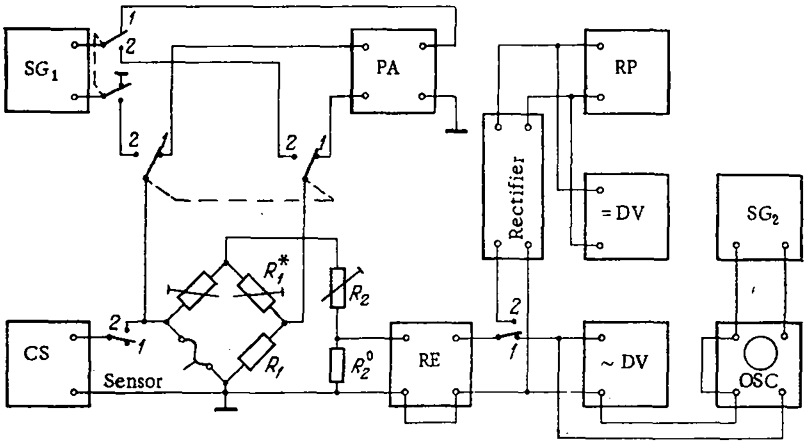

Fig. 1. Block diagram of the experimental set-up: SG, acoustic-frequency signal generator; RA, resonance amplifier; PA, power amplifier; CS, dc source; OSC, oscilloscope; ~DV and =DV, ac and dc digital voltmeters; RP, recording potentiometer. Switch positions: 1, regular thermal-regime method; 2, cylindrical-probe method.

The method of the regular thermal regime of the third kind is based on the measurement of the temperature fluctuation of a temperature sensor with small inertia when it is heated by alternating current [1]. The fluctuation amplitude, other conditions being equal, depends on the coefficient of thermal activity of the medium surrounding the sensor. The thickness of the investigated layer is equal to the thermal wavelength

$$\Lambda = \sqrt{2\pi a/f}. \tag{1}$$

By changing the frequency of the input voltage we can therefore change the thickness of the investigated layer of the medium near the sensor. For the reinforced composite material this method makes it possible to measure the thermal activity of the contact zone and to find the boundaries of the zone.

The cylindrical-probe method is based on the analytical description of the temperature field around an infinitely long linear heat source in an infinite medium [2]. The heating of the probe above the initial temperature depends on the thermophysical properties of the surrounding medium and on the contact thermal resistance between the probe and medium. If we assume that the probe has an ideal thermal contact with the surrounding medium, we can find the coefficients of thermal conductivity and thermal diffusivity of the material by this method. If we know the thermal constants of the material we can determine the contact thermal resistance between the probe and the material, and in combination with the previous method we can determine the thermal constants of the contact zone near the probe.

Experimental Method

The block diagram of the experimental set-up is shown in Fig. 1. The sensor is included in one branch of the ac bridge.

When the set-up works in the regular thermal regime of the third kind the measuring bridge receives alternating voltage from the signal generator SG via the power amplifier PA. Because of the temperature fluctuations, the sensor resistance changes and voltage appears across the bridge. This voltage is proportional to the magnitude of the temperature fluctuation [1]:

$$e = \frac{1}{2}\,\frac{ER_1\bar{r}\beta|\vartheta|}{(R_1+\bar{r})^2}\,[\cos(\omega\tau-\varphi)+\cos(3\omega\tau-\varphi)]. \tag{2}$$

The voltage across the bridge contains a signal of the fundamental frequency and a signal with three times this frequency. The treble-frequency signal is separated and amplified by the resonance amplifier RA and is measured by the ac digital voltmeter ~DV.

The coefficient of thermal activity of the surrounding medium is determined from an equation containing the temperature fluctuation $|\vartheta|$ [1]:

TABLE 1. Results of the Control Experiments with Glycerine

f, Hz	$\bar{b}$, W·sec$^{1/2}$/m^2·deg	$\sigma_{\bar{b}}$	$\bar{b}\pm t\sigma_{\bar{b}}$	$\bar{\lambda}$, W/m·deg	$\bar{\lambda}\pm t\sigma_{\bar{\lambda}}$	F_{ch}	$(F_{cr})_{0,95}$
20,0	990,9	3,0	984—998	0,289	0,285—0,292		
40,7	988,0	4,5	977—991	0,285	0,281—0,289	1,24	3,72
73,5	998,0	13,5	963—1025	0,292	0,274—0,310		

$$2b^2 + 2bd + d^2 = \left(\frac{W}{|\vartheta| S\sqrt{\omega}}\right)^2 = \left(\frac{k}{y}\right)^2, \tag{3}$$

where

$$d = \frac{2\sqrt{\bar{\omega}}\, c_s m}{S}; \quad k = \frac{\beta k_{RA} R_1}{4 S_c \sqrt{\omega}\, V}; \quad y = \frac{(R_2/R_2^0 + 1)}{E^3} k_1 k_2;$$

$$k_1 = \frac{(R_1 + \bar{r})^4}{\bar{r}^2}; \quad k_2 = \frac{S}{S_c} = \frac{r_0}{(r_0)_c}.$$

The coefficients d and k in Eq. (3) can be calculated from the expressions given above, but it is better to determine them by a calibration experiment with two calibrated materials whose thermophysical properties are known. In this experiment, a calibrated sensor should be used since the length of the working sensors can be different. The difference between the lengths of the calibrated and working sensors is taken into account by including the coefficient k_2.

In the cylindrical-probe regime, two input sources are used. This is because here the probe performs all the required functions, in contrast with the usual probe construction which consists of three elements (namely the probe, heater, and the temperature sensor). The heating is done by the dc current and the magnitude of the excess probe temperature is measured by the small variable voltage appearing at the bridge. Since the probe resistance changes with heating, a signal appears due to the bridge being off balance. This signal is proportional to the excess temperature of the probe and is amplified by the resonance amplifier RA, rectified and recorded by the potentiometer RP. Using this recording we can determine the excess temperature of the probe at two instances τ_1 and τ_2 in the quasistationary thermal regime. The thermal constants of the material are determined from the expressions

$$\lambda = \frac{k_3 k_4}{(y_2 - y_1)} \ln(N), \tag{4}$$

$$a = \frac{k_5}{\tau_1} \exp\left[\frac{y_1}{y_2 - y_1} \ln(N) - k_6\lambda\right], \tag{5}$$

where

$$N = \tau_2/\tau_1, \; y_1 = k_4\theta(\tau_1), \; y_2 = k_4\theta(\tau_2),$$

$$k_3 = \frac{W}{4\pi L}; \quad k_4 = \frac{R_1 r_0 \beta k_{RA}}{(R_1 + r_0)^2}; \quad k_5 = \frac{D\exp(\gamma)}{16}; \quad k_6 = \frac{4}{\alpha D}.$$

Results of Control Experiments

To find the accuracy of the experimental set-up we carried out control experiments with calibrated liquids (toluene, distilled water, propyl alcohol, and glycerine). Two liquids were used to calibrate the set-up, and the other two were used to find the accuracy of the experiment. For the sensor we used a silver wire of diameter 0.15 mm and electric resistance 0.1-0.2 Ω.

Some result of the experiment for glycerine, statistically processed, are shown in Table 1. Several measurements were made for each of the three frequencies of the input voltage. The average value of a quantity b, for example, was determined from the expression

$$\bar{b} = \frac{1}{n}\Sigma b_i. \tag{6}$$

TABLE 2. Results of the Experiments with Samples from Reinforced Composite Material

	f, Hz	Sample number					
		1	2	3	4	5	6
b_{cz}, $W \cdot sec^{1/2}/m^2 \cdot deg$	20,0	839	875	901	878	941	932
	40,7	916	909	913	947	976	983
λ_{cz}, W/m·deg	20,0	0,362	0,394	0,418	0,395	0,459	0,445
	40,7	0,432	0,425	0,429	0,462	0,490	0,497

The root-mean-square deviation of b_i from the average $\bar{b}$ was found from

$$\sigma_{\bar{b}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (b_i - \bar{b})^2}. \tag{7}$$

The confidence interval of the average values was determined from

$$P(\bar{b} - t\sigma_{\bar{b}} \leqslant b \leqslant \bar{b} + t\sigma_{\bar{b}}) = P_{cp}, \tag{8}$$

where P_{cp} is the confidence probability and t is a parameter found from the tables of the Student distribution [3].

To determine the effect of frequency on the experimental results we performed a dispersion analysis following the method described in [4]. Using the Fisher–Snedecor F test [3] we compared the factor and residual dispersion. If $F_{ch} > F_{cr}$ for the experimentally determined quantities, where F_{cr} is the critical value of the F test found from tables of the F distribution [3], the effect of frequency on the experimental results is significant.

The control experiments make it possible to make the following conclusions:

1) The experimental set-up is reasonably accurate even for the very small resistance of the sensor (0.1 – 0.2 Ω).

2) The thermal constants are independent of frequency ($F_{ch} < F_{cr}$) as expected since the properties of homogeneous liquids are independent of the depth of the layer under investigation. Large systematic errors are therefore unlikely.

Results of the Experiment with the Composite Material

On the described experimental set-up we conducted a series of experiments to determine the thermal constants of the contact zone in samples from the composite material reinforced by a silver wire of diameter 0.15 mm. Some results of the experiment are shown in Table 2. It is seen that there are differences in the thermal constants for each of the samples at different frequencies, and between samples at one frequency of the input voltage. The dispersion analysis shows that both the frequency and sample factors are significant. From the result of the experiment we can make the following conclusions:

1) The thermal constants of the contact zone are different from those of the composite material with $\lambda = 0.33\ W \cdot m^{-1}\ deg^{-1}$ and $b = 800\ W \cdot sec^{1/2} \cdot m^{-2} \cdot deg^{-1}$. With decreasing thickness of the layer the quantities λ_{cz} and b_{cz} decrease.

2) The thermal constants are different even for samples from the same batch. It is seen, therefore, that the properties of the contact zone are influenced by technological factors which are not controlled by the conditions of manufacture of the reinforced products.

NOTATION

λ and a, thermal conductivity and thermal diffusivity; b, thermal activity; c, specific heat; e, bridge off-balance voltage; E and f, the voltage and frequency of the alternating current supplied to the bridge; ω, angular frequency; φ, off-balance signal phase shift relative to the input voltage; τ, time; r, S, and m, electric resistance, contact surface area and mass of the probe; $\bar{r}$, electric resistance at the mean heating temperature $\bar{T}$; β, temperature resistance coefficient; W, electric power; $|\vartheta|$, probe temperature fluctuation relative to the mean $\bar{T}$; $\Theta(\tau) = T - T_0$, excess temperature of the probe relative to the initial temperature T_0; k_{RA}, amplification coefficient of RA; R_1, standard resistance in the bridge; R_2 and R_2^0, resistances of the potential divider at the RA input; V, voltage at the output of RA; D and L, diameter and length of the probe; $1/\alpha$, thermal resistance; $\gamma = 0.577215$, Euler's constant; and $\pi = 3.1415926$. Indices: 0, at the initial temperature T_0; s, referring to the sensor; c, to the calibrated sensor; and cz, to the contact zone.

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NUMERICAL SOLUTION OF THE INVERSE HEAT-CONDUCTION PROBLEM FOR DETERMINING THERMAL CONSTANTS

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We investigate the solution of the inverse heat-conduction problem for a cylinder, based on a series expansion of a thermal constant in powers of temperature, and the determination of the series coefficients by a direct-search method.

The majority of experimental methods of determination of the thermal constants of solid materials is based on the solution of the linear or nonlinear heat equation with some specific boundary conditions [1]. The use of these methods is brought about by the necessity of ensuring a stationary thermal regime, and monotonic or instantaneous heating to the required temperature which presents appreciable difficulties. In recent years it has been preferred to determine the thermal constants by the numerical solution of the inverse heat-conduction problem [2-7]. These methods do not as a rule, impose any restriction on the change of the boundary conditions. The physical parameter which appears in the heat equation is found in this case from the known boundary conditions and from the temperature at interior points.

In the present work we investigate a numerical solution of the inverse heat-conduction problem which can be immediately used for the experimental determination of the thermal conductivity or some other constant which appears in the heat equation. The solution is based on a series expansion of the required thermal constant in a series in powers of temperature and on the determination of the series coefficients by a specially derived method of direct search. We note that this method can be used for the solution of any one-dimensional heat-conduction problem with coefficients of interest, with any boundary conditions.

The problem of determination of a thermal constant from the experimentally measured values of temperature at two points of a sufficiently long, hollow, or dense cylinder can be represented by the following equations:

$$c\rho \frac{\partial t}{\partial \tau} = \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r\lambda \frac{\partial t}{\partial r}\right), \quad r_0 < r < R, \quad 0 < \tau < \tau_F, \tag{1}$$

$$t(r,\ 0) = \varphi(r), \tag{2}$$

$$t(R,\ \tau) = \psi(\tau), \tag{3}$$

$$\frac{\partial t(r_0,\ \tau)}{\partial r} = 0. \tag{4}$$

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